International Journal of Theoretical Physics, Vol. 40, No. 2, 2001

Long-Range Behavior of Dual-Restricted QCD Vacuum

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Received June 10, 1999

We analyze the geometrical structure of the local non-Abelian gauge theory in terms of the magnetic symmetry, using the resemblance between the non-Abelian gauge formulations and Einstein's theory of gravitation in a higher dimensional unified space. The mathematical foundation of dual QCD in fiber bundle form is then discussed and used for the analysis of the important problem of color confinement in QCD. The associated Lagrangian formulation in magnetic gauge is shown to lead to dual dynamics due to the emergence of the topological charges of magnetic nature. The dynamical breaking of magnetic symmetry is shown to lead to the magnetic condensation of the QCD vacuum. A state of the dual superconductivity in the QCD vacuum is then shown to evolve which ultimately pushes the QCD vacuum to the confining phase. The flux tube structure of the magnetically condensed QCD vacuum is analyzed by computing the asymptotic string solutions of the field equations. The energy content of such confining structures is computed and analyzed in terms of its logarithmic and linear nature.

1. INTRODUCTION

Quantum chromodynamics is a non-Abelian gauge theory of strong interactions and exhibits many interesting nonperturbative phenomena, e.g., quark confinement, chiral symmetry breakdown, mass spectrum of the physical states, etc. However, its precise physical meaning has been elusive. The issue of quark confinement is one of the main problems of hadronic physics that needs to be resolved for the correct understanding of the structure of hadrons as well as for quark-nuclear physics, which is basically governed by the QCD dynamics. There has been great progress in the theoretical study of these nonperturbative phenomena due to the understanding of the dual Ginzburg–Landau (DGL) theory [12] and the lattice gauge theory [14]. Taking

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0020-7748/01/0200-0477$19.50/0

the analogy of superconductivity, Nambu [18] and others [10, 27] have shown that the color confinement in dual QCD occurs in a similar way as the magnetic flux confinement occurs in a superconductor due to the Meissner effect. Further, as proposed by 't Hooft [10], the SU(N) gauge theory can be reduced to $U(1)^{N_c-1}$ gauge theory with monopole by Abelian gauge fixing. In this gauge, the QCD monopole appears as a topological object whose condensation may lead to the color confinement through the dual Meissner effect [16, 17, 27]. Therefore, the dual Meissner effect, with the role of the (chromo) electric and (chromo) magnetic fields reversed, may be an appealing explanation for the color confinement. The chromoelectric field between the color isocharges is expelled from the embedding vacuum and the chromoelectric flux tube generates a linear rise in the energy [1, 29, 30] among quark pairs. It is therefore expected to be responsible for the large-distance confining behavior of the QCD vacuum through the condensation of magnetically charged objects [10, 21].

As it is well known that the non-Abelian gauge theories essentially exhibit an inherent built-in duality, a number of attempts have been made recently to formulate QCD as a dual gauge theory [2, 3, 6, 22, 23, 27]. Imposing the magnetic symmetry, a restricted version of dual QCD has been formulated [4, 5, 20] and used for the explanation of confinement mechanism. In the present paper, the geometrical structure of the local non-Abelian gauge theory is used to explain the problem of color confinement. Constructing the Lagrangian for such dual theory in magnetic gauge, we show that the dynamical breaking of the magnetic symmetry by an effective potential creates a state of dual superconductivity, thereby pushing the QCD vacuum to the confining phase. The field equations associated with the flux tube structure in the confining phase are derived. Finally, the long-range behavior of the dual RCD vacuum is analyzed by computing the asymptotic string solutions and the finite energy per string length particularly for large-scale considerations.

2. MAGNETIC SYMMETRY AND DUAL DYNAMICS IN NON-ABELIAN GAUGE THEORY

The non-Abelian theory of gauge fields may be viewed as the Einstein theory of gravitation in a higher dimensional unified space and allows the introduction of some additional internal symmetries [7, 22, 24, 28]. In this connection, the introduction of the magnetic symmetry in such non-Abelian formulations of QCD plays an important role and has important bearings on the dual dynamics of the theory. In such a higher dimensional metric formulation of the gauge theory, the unified space P consists of the four-dimensional external space M and n-dimensional internal group space G and is identified as the $(4 + n)$-dimensional metric manifold of g_{AB} ($A, B = 0, 1, \ldots, 3 + n$). Then, P involves an n-dimensional isometry group with n-Killing vector fields ξ_i ($i = 1, 2, \ldots, n$), which satisfy the canonical commutation relations given by

$$[\xi_i, \xi_j] = f_{ij}{}^k \xi_k \tag{2.1}$$

and the Killing condition given by

$$\pounds_{\xi_i} g_{AB} = 0 \tag{2.2}$$

where $\pounds_{\xi_i}$ is the Lie derivative along the direction of ξ_i. Since these Killing vector fields may be viewed as an n-dimensional involutive distribution on P, they admit a unique maximal integral manifold as a metric submanifold, i.e., the internal metric,

$$\phi_{ij} = g_{AB} \xi_i^A \xi_j^B \tag{2.3}$$

is invertible. The unified space may be regarded as a principal fiber bundle $P(M, G)$ over space-time if we identify the quotient space P/G as the base manifold M with a canonical projection Π: $P \to M$. As such, with the choice of the vector fields ξ_i as orthonormal ones and G as a semi-simple group, the metric ϕ_{ij} becomes of the topologically meaningful Cartan–Killing form. The corresponding Einstein theory in unified space then becomes the canonical Yang–Mills theory. One can now define the magnetic symmetry as an additional internal isometry H, which forms a subgroup of the structure group G which commutes with it and admits some additional Killing vector fields. Let one of the Killing vector fields be m; then by assumption, we have

$$m = m^i \xi_i, \qquad [m, \xi_i] = 0 \tag{2.4}$$

and for the associated Lie derivative

$$\pounds_m g_{AB} = 0 \tag{2.5}$$

It follows that the multiplet $\hat{m}$ can be written as

$$\hat{m} = [m^1, m^2, \ldots, m^n]^T \tag{2.6}$$

(T denoting the transpose), which must form an adjoint representation of the gauge group with the internal metric ϕ_{ij} chosen to be of the Cartan–Killing form. The Killing condition (2.5) restricts the metric ϕ_{ij} as well as the associated potential (connection), and can be written as

$$D_\mu \hat{m} = 0, \qquad \text{i.e.,} \quad \partial_\mu \hat{m} + g\vec{W}_\mu \times \hat{m} = 0 \tag{2.7}$$

where $\vec{W}_\mu$ is the gauge potential of the group G. Thus, the magnetic symmetry restricts the connection to those whose holonomy bundle becomes a reduced bundle $P(M, H)$. It therefore restricts the dynamical degree of freedom of the theory, while keeping full gauge degrees of freedom intact, and hence the connection (potential) satisfying the above Killing condition is of restricted nature. It is clear that the magnetic symmetry associated with $\hat{m}$ clearly imposes strong constraints on connection and may be regarded as the symmetry of the potentials. The connection, which satisfies the Killing condition, is called the restricted connection, and for SU(2), the restricted potential [choosing $G \equiv \mathrm{SU}(2)$ and $H \equiv \mathrm{U}(1)$] may be obtained as

$$\vec{W}_\mu = A_\mu \hat{m} - \frac{1}{g}\,\hat{m} \times \partial_\mu \hat{m} \tag{2.8}$$

Here, $\hat{m} \cdot \vec{W}_\mu \equiv A_\mu$ is the color electric potential of the Cartan subgroup and is not restricted by the constraint. On the other hand, the second part on the right-hand side of Eq. (2.8) is completely determined by the magnetic symmetry and is topological (dual magnetic) in nature. Thus, the beauty of the magnetic symmetry is that it can be used to describe the topological structure of the gauge symmetry and the multiplet $\hat{m}$ may then be viewed as defining the homotopy of the mapping $\Pi_2(S^2)$,

$$\hat{m}: \quad S_R^2 \rightarrow S^2 = \mathrm{SU}(2)/\mathrm{U}(1) \tag{2.9}$$

where S_R^2 is the two-dimensional sphere of the three-dimensional space and S^2 is the group coset space fixed by $\hat{m}$. It shows clearly that by imposing magnetic symmetry on the potential, one may bring the topological structure into the dynamics explicitly.

The field strength $\vec{G}_{\mu\nu}$ corresponding to the potential (2.8) may be constructed in the form

$$\begin{aligned}\vec{G}_{\mu\nu} &= \partial_\mu \vec{W}_\nu - \partial_\nu \vec{W}_\mu + g\vec{W}_\mu \times \vec{W}_\nu \\ &= (F_{\mu\nu} + B_{\mu\nu})\hat{m}\end{aligned} \tag{2.10}$$

It satisfies the identity given by

$$\vec{G}_{\mu\nu} \times \hat{m} = \frac{1}{g}\,[D_\mu, D_\nu]\hat{m} \tag{2.11}$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \tag{2.12}$$

$$B_{\mu\nu} = -\frac{1}{g}\,\hat{m} \cdot (\partial_\mu \hat{m} \times \partial_\nu \hat{m}) \tag{2.13}$$

which holds for any arbitrary G. Thus, the total gauge field strength takes separate contributions from both parts, one unrestricted, and the other completely determined by the magnetic symmetry. It reflects the dual structure of the formulation, which already appeared at the level of the potential (2.8) in a general way independent of the choice of the symmetry G. Since, the magnetic charge is topological in origin and the part $B_{\mu\nu}$ is determined completely by the topological degrees of freedom of the theory, this part of the field strength can be identified as the magnetic one. It leads to the perfect dual structure at the field strength level with the general non-Abelian field strength given as

$$\vec{G}_{\mu\nu} = \vec{F}_{\mu\nu} + \vec{B}_{\mu\nu} \tag{2.14}$$

The above dual structure becomes more interesting when the topological structure is brought further into the dynamics explicitly. So, for the SU(2) case, let us rotate the magnetic vectors $\hat{m}$ to a pre-fixed spacetime-independent direction (say ξ_3 in isospace) by a gauge transformation

$$\hat{m} \xrightarrow{U} \xi_3 = U\hat{m} = [0, 0, 1]^T \tag{2.15}$$

with the parameterization

$$\hat{m} = \begin{bmatrix} \sin\alpha & \cos\beta \\ \sin\alpha & \sin\beta \\ \cos\alpha & \end{bmatrix} \tag{2.16}$$

Choosing $U = \exp(-\alpha t_2 - \beta t_3)$ in accordance with Eq. (2.15), one obtains in the magnetic gauge

$$\vec{W}_\mu \xrightarrow{U} \vec{W}'_\mu = (A_\mu + B^*_\mu)\hat{\xi}_3 \tag{2.17}$$

where

$$\vec{B}^*_\mu = B^*_\mu \hat{m} = \frac{1}{g}\cos\alpha\ \partial_\mu\beta\ \hat{m} \tag{2.18}$$

The associated field strength (in magnetic gauge) is also given by

$$\vec{G}_{\mu\nu} \xrightarrow{U} \vec{G}'_{\mu\nu} = (F_{\mu\nu} + B_{\mu\nu})\hat{\xi}_3 \tag{2.19}$$

The part associated with the topological degrees of freedom is then given by

$$B_{\mu\nu} = -\frac{1}{g}\,\mathrm{Sin}\,\alpha(\partial_\mu\alpha\;\partial_\nu\beta - \partial_\nu\alpha\;\partial_\mu\beta) \tag{2.20}$$

which is expressible in the form of the dual potential as

$$B_{\mu\nu} = B^*_{\nu,\mu} - B^*_{\mu,\nu} \tag{2.21}$$

Hence, the potential B^*_μ can be identified as the magnetic potential associated with the topological monopole, which is completely determined by $\hat{m}$ up to the Abelian magnetic gauge degrees of freedom. Consequently, in the magnetic gauge, one may indeed bring the topological (magnetic) properties of $\hat{m}$ down to the dynamical variable B_μ by removing all nonessential gauge degrees of freedom. The important feature of the additional internal symmetry discussed here is that, while keeping the full gauge degrees of freedom intact, it restricts and reduces the dynamical degrees of freedom and provides a self-consistent nontrivial subset of the original gauge theory. The resulting theory may therefore be identified as a restricted dual gauge theory (restricted chromodynamics).

3. FIELD EQUATIONS ASSOCIATED WITH FLUX TUBE IN DUAL RCD

In this section, we formulate the flux tube system in dual RCD taking the analogy from the dual Ginzburg–Landau (DGL) theory. The field-theoretical formulation for such dual gauge theory may be developed explicitly using the potential $\vec{W}_\mu$. For the simple case of SU(2), the QCD Lagrangian can then be written as

$$L = -\frac{1}{4}\vec{G}^2_{\mu\nu} + \bar{\psi} i\gamma^\mu D_\mu \psi - m_0\bar{\psi}\psi \tag{3.1}$$

where ψ is the quark doublet and $\vec{G}_{\mu\nu}$ is the gauge field strength, which for the RCD can be obtained by substituting the unrestricted potential by restricted one, viz. Eq. (2.8). Here, the Lagrangian obtained by this substitution has some undesirable features, one of which is that the monopole appears as a pointlike object, but not as a regular field. Second, the magnetic potential B^*_μ of the monopole describes the magnetic field of the monopole by a spacelike potential and contains the well-known string singularity. By introducing the dual magnetic potential B_μ, which can describe the magnetic field of the monopole with a regular timelike potential, and at the same time a complex scalar field ϕ for the monopole, one can remove these undesirable features. By using the dual gauge field, the RCD Lagrangian leads to the following modified form:

$$L_{dr}^{\mathrm{m}} = \bar{a}i\gamma^{\mu}\left(\partial_{\mu} + \frac{g}{2i}(A_{\mu} + B_{\mu}^{*})\right)a + \bar{b}i\gamma^{\mu}\left(\partial_{\mu} - \frac{g}{2i}(A_{\mu} + B_{\mu}^{*})\right)b$$
$$+ m_0(\bar{a}a + \bar{b}b) - \frac{1}{4}F_{\mu\nu}^2 - \frac{1}{2}F_{\mu\nu}B^{\mu\nu} - \frac{1}{4}B_{\mu\nu}^{*2}$$
$$+ \left|\left((\partial_{\mu} + i\frac{4\pi}{g}(A_{\mu}^{*} + B_{\mu})\right)\phi\right|^2 - V(\phi^{*}\phi) \tag{3.2}$$

where a and b are red and blue quarks, A_{μ} and B_{μ} are the regular potentials which describe the (color) electric and magnetic charges with the timelike potentials, and

$$F_{\mu\nu} = A_{\nu,\mu} - A_{\mu,\nu} \tag{3.3}$$

$$B_{\mu\nu}^{*} = \frac{1}{2}\varepsilon_{\mu\nu\sigma\rho}B^{\rho\sigma} = B_{\nu,\mu} - B_{\mu,\nu} \tag{3.4}$$

The Lagrangian given by Eq. (3.2) is an effective Lagrangian for dual dynamics of RCD at the phenomenological level and has some interesting features. It describes the possibility of the occurrence of two phases in such a theory. One is the deconfinement phase, where magnetic symmetry is preserved, and the other is the confinement phase, where the magnetic symmetry is indeed broken dynamically. In the first phase, not only the quarks, but the monopoles also appear as physical particle states. On the other hand, in the confinement phase, they disappear from the physical spectrum and the theory is expected to contain two magnetic glueballs as scalar and vector modes of the condensed vacuum.

For the explanation of confinement mechanism of the colored sources explicitly, let us write the Lagrangian (3.2) in the absence of quarks and introduce the monopole source by a complex scalar field ϕ (identified as the order parameter). With these considerations, we have

$$L_{dr}^{\mathrm{m}} = -\frac{1}{4}B_{\mu\nu}^{*2} + \left|\left(\partial_{\mu} + i\frac{4\pi}{g}B_{\mu}\right)\phi\right|^2 - V(\phi^{*}\phi) \tag{3.5}$$

This Lagrangian is known to generate the dynamical symmetry breaking through the effective potential. It, in turn, leads to the magnetic condensation of the vacuum and guarantees the dual Meissner effect, which confines any color electric flux present. The effective potential responsible for the dynamical magnetic symmetry breaking is fixed by the requirement of ultraviolet finiteness and infrared instability of the dual RCD Lagrangian. As obtained by Coleman and Weinberg [8] by using the single-loop expansion technique, it has the form given by

$$V_{\text{eff}} = \frac{24\pi^2}{g^4}\left[\phi_0^4 + (\phi^*\phi)^2\left(2\ \ln\frac{\phi^*\phi}{\phi_0^2} - 1\right)\right] \tag{3.6}$$

where $\phi_0 = \langle\phi^*\phi\rangle^{1/2}$ is the expectation value of ϕ. With such a Lagrangian, the field equations in the magnetically condensed vacuum are obtained in the form

$$\left(\partial^\mu - i\,\frac{4\pi}{g}\,B^\mu\right)\left(\partial_\mu + i\,\frac{4\pi}{g}\,B_\mu\right)\phi - \frac{24\pi^2}{g^4}\left(4\phi\phi^*\ \ln\frac{\phi\phi^*}{\phi_0^2}\right)\phi = 0 \tag{3.7}$$

$$\partial^\nu B^*_{\mu\nu} + i\,\frac{4\pi}{g}\,(\phi^*\overleftrightarrow{\partial}_\mu\phi) - \frac{32\pi^2}{g^2}\,B_\mu\phi\phi^* = 0 \tag{3.8}$$

where $\phi^*\overleftrightarrow{\partial}_\mu\phi = \phi\partial_\mu\phi^* - \phi^*\partial_\mu\phi$.

The unusual features of the dual RCD vacuum responsible for quark confinement may become more transparent if one starts with the Nielsen and Olesen [19] interpretation of vortex line solutions, so that the monopole pairs can exist inside the superconductor in the form of thin flux tubes leading to the confinement of colored fluxes. Orienting the flux tubes inside the hadronic sphere along the direction of the z-axis and using the cylindrical coordinates (ρ, θ, z), we can write the potentials as

$$\begin{aligned} B^1 &= B_\rho \cos\theta - B_\theta \sin\theta, \qquad & B^2 &= B_\rho \sin\theta + B_\theta \cos\theta \\ B^3 &= B_z, & B^0 &= B_t \end{aligned} \tag{3.9}$$

The potential (B^*_μ) appearing in the field strength tensor $B_{\mu\nu}$ is given by Eq. (2.18) and one can replace $B^2_{\mu\nu}$ by $B^{*2}_{\mu\nu}$ with no change of signature in Eq. (2.11). At first glance, this replacement appears to be wrong. However, it seems to be correct in view of the fact that the correct signature is obtained by requiring that the Hamiltonian of the theory should remain the same while one switches over to the regular potential B_μ. Hence, using the cylindrical symmetry, we can write

$$B_\mu = \frac{1}{g}\cos\alpha\ \partial_\mu\beta$$

which implies

$$B_\rho = \frac{1}{g}\cos\alpha\ \partial_\rho\beta \tag{3.10a}$$

$$B_\theta = \frac{1}{g}\cos\alpha\ \frac{1}{\rho}\ \partial_\theta\beta \tag{3.10b}$$

$$B_z = \frac{1}{g} \cos \alpha \ \partial_z \beta \tag{3.10c}$$

For $\phi(x)$, we use the ansatz given by

$$\phi(x) = \exp(in\theta)\chi(\rho) \qquad (n = 0, \pm 1, \pm 2, \ldots) \tag{3.11}$$

In view of the uniqueness of the function $\phi(x)$, we have

$$B_\theta(x) = B(\rho), \qquad \text{as} \qquad B_t = B_\rho = B_z = 0 \tag{3.12}$$

Let us now take $\vec{E}_m$ and $\vec{B}$ with z and θ components, respectively. This leads to

$$B = B(\rho) = \frac{1}{g} \cos \alpha \ \frac{1}{\rho} \ \partial_\theta \beta \tag{3.13}$$

The color electric induction $E_m(\rho)$ along the z direction has the form

$$E_m\,(\rho) = -\frac{1}{\rho}\frac{d}{d\rho}\,[\rho B(\rho)] \tag{3.14}$$

where we use $\vec{E}_m = \vec{\nabla} \times \vec{B}$ with $B(\rho) = B_\theta = B$.

The equations of motion in terms of cylindrical coordinates and the cylindrical symmetric potentials may be derived in the form

$$\frac{d}{d\rho}\left[\frac{1}{\rho}\frac{d}{d\rho}\left(\frac{1}{g}\cos \alpha \partial_\theta \beta\right)\right] + \frac{8\pi}{g}\left(\frac{n}{\rho} + \frac{4\pi}{g^2\rho}\cos \alpha \partial_\theta \beta\right)\chi^2(\rho) = 0 \tag{3.15}$$

$$\frac{1}{\rho}\frac{d}{d\rho}\left(\rho\,\frac{d\chi(\rho)}{d\rho}\right) - \left[\left(\frac{n}{\rho} + \frac{4\pi}{g^2\rho}\cos \alpha\ \partial_\theta\beta\right)^2 + \frac{24\pi^2}{g^4}\left(4\chi(\rho)^2 \ln \frac{\chi^2(\rho)}{\phi_0^2}\right)\right]\chi(\rho) = 0 \tag{3.16}$$

Using Eqs. (3.10) and (3.11), one can directly write down these equations in the simplest form as

$$\frac{d}{d\rho}\left[\frac{1}{\rho}\frac{d}{d\rho}\,(\rho B(\rho))\right] - \frac{8\pi}{g}\left(\frac{n}{\rho} + \frac{4\pi}{g}\,B(\rho)\right)\chi^2(\rho) = 0 \tag{3.17}$$

$$\frac{1}{\rho}\frac{d}{d\rho}\left(\rho\,\frac{d\chi(\rho)}{d\rho}\right) - \left[\left(\frac{n}{\rho} + \frac{4\pi}{g}\,B(\rho)\right)^2 + \frac{24\pi^2}{g^4}\left(4\chi^2(\rho)\,\ln\,\frac{\chi^2(\rho)}{\phi_0^2}\right)\right]\chi(\rho) = 0 \tag{3.18}$$

4. CLASSICAL VACUUM SOLUTIONS AND FIELD ENERGY

Using the Lagrangian density given by Eq. (3.5), we can find the energy per string length (the string tension). For the static case ($B_0 = 0$), it has the form

$$K = \int_0^\infty \rho \, d\rho(-L_{dr}^m)$$

$$= 2\pi \int_0^\infty \rho \, d\rho \left[\frac{1}{2\rho^2} \left\{ \frac{d}{d\rho} \rho B(\rho) \right\}^2 + \left(\frac{d}{d\rho} \chi(\rho) \right)^2 + \left(\frac{4\pi}{g} B(\rho) + \frac{n}{\rho} \right)^2 \chi^2(\rho) + \frac{24\pi^2}{g^4} \left(\phi_0^4 + (\chi(\rho))^4 \left(2 \ln \frac{\chi^2(\rho)}{\phi_0^2} - 1 \right) \right) \right] \tag{4.1}$$

The simplest solution that minimizes the energy is given by

$$B(\rho) = -\frac{ng}{4\pi\rho} \tag{4.2}$$

and

$$\chi(\rho) = \phi_0 \tag{4.3}$$

For these solutions, the energy K has the lowest value and they are referred to as the classical vacuum solutions satisfying the equations of motion. The field equations given by Eqs. (3.17) and (3.18) are a complicated set of coupled nonlinear differential equations and we do not have any exact solution for them. However, one can obtain the associated asymptotic solution taking the variation for $B(\rho)$ as

$$B(\rho) = -\frac{ng}{4\pi\rho} [1 + F(\rho)] \tag{4.4}$$

Using the above variation with the equations of motion and taking the approximation for large ρ as $\chi \underset{\rho\to\infty}{\to} |\phi|$, we find the asymptotic behavior for large values of ρ,

$$F(\rho) \underset{\rho\to\infty}{\to} C\rho^{1/2} \exp\left(-\frac{4\pi}{g} \sqrt{2} \, \phi_0\rho \right) \qquad (C = \text{const}). \tag{4.5}$$

The above solution for an infinitely long RCD flux tube is similar to the Nielson–Olesen [19] magnetic vortex solution for the vector potential and shows that the vector potential vanishes exponentially at large distances. Utilizing such asymptotic behavior of the associated dual QCD fields, we find the string energy for large ρ,

$$K = D(\alpha_s A^2 + 8\pi\phi_0^2) \int_0^\infty \exp(-2A\rho) \, d\rho - AD \, \alpha_s \int_0^\infty \frac{\exp(-2A\,\rho)}{\rho} \, d\rho$$
$$= K_1 + K_2 \tag{4.6}$$

with

$$K_1 = D(\alpha_s A^2 + 8\pi\phi_0^2) \int_0^\infty \exp(-2A\rho)\, d\rho$$

$$K_2 = -AD\, \alpha_s \int_0^\infty \frac{\exp(-2A\rho)}{\rho}\, d\rho$$

The constants D and A are given as

$$A = \frac{4\pi\sqrt{2}\phi_0}{g} \qquad \text{and} \qquad D = \frac{n^2c^2}{4}$$

and α_s $(= g^2/4\pi)$ is the fine structure constant of dual chromodynamics. (Here we have dropped the terms with higher powers of ρ in the denominator due to the rapidly decreasing nature of the integral.)

Let us now analyze the behavior of these integrals separately.

1. The first part of K (viz. K_1) clearly shows the decaying behavior for large values of ρ and is therefore dominant only for the small values of ρ.

2. The second integral may be analyzed after decomposing it into two parts,

$$K_2 = K^{\mathrm{d}} + K^{\mathrm{c}} \tag{4.7}$$

where

$$K^{\mathrm{d}} = -AD\, \alpha_s \int_0^R \frac{\exp(-2A\rho)}{\rho}\, d\rho$$

$$K^{\mathrm{c}} = -AD\, \alpha_s \int_R^\infty \frac{\exp(-2A\rho)}{\rho}\, d\rho$$

(here, we have introduced the parameter R having its maximum value as the radius of the hadronic sphere, and the flux tube is taken in such a way that the maximum height or the length of the tube is the diameter of the hadronic sphere). The second part of the integral (viz. K^{c}) may be evaluated by using the incomplete gamma function with exponential integral and taking the series expansion in the form

$$K^{\mathrm{c}} = AD\, \alpha_s \left(\gamma + \ln R + \sum_{n=1}^{\infty} \frac{(-2A)^n R^n}{n \cdot n!} \right) \tag{4.8}$$

where γ is the Euler–Maclaurin constant. This expression shows that the energy needed to liberate a quark from a color singlet is proportional to the power of its distance of separation, and the expression then leads to a power law for the confinement. Thus, the expression given by Eq. (4.8) shows that

$$K^{c} \propto \alpha_s R^n \qquad (\text{with } n \geq 1)$$

For $n > 1$, the energy between the quarks increases with the separation, whereas for $n = 1$, it becomes linear in the separation. Thus, for $n = 1$, the color sources in the system are necessarily confined permanently. Permanent quark confinement (for a given value of α_s) therefore, requires a potential having $n \geq 1$.

The string energy becomes more transparent if we introduce the critical radius R_c of the phase transition. It leads to two different situations. In the case when $R > R_c$, the integral K^c becomes dominant and yields the dominant logarithmic as well as linear contributions in powers of R for the confinement potential, and hence the whole QCD vacuum goes over to the strong confining phase. On the other hand, for the case when $R < R_c$, the integral K^d becomes dominant, which ultimately pushes the theory to the normal (i.e., deconfining) phase.

5. CONCLUSIONS

Using the dual gauge theory of QCD, we have analyzed the flux tube system between quark pairs. The dual symmetric restricted gauge potential constructed in terms of magnetic vectors on global sections describes the dual dynamics associated with non-Abelian monopoles. The dual magnetic potential derived in terms of the Eq. (3.10) in magnetic gauge is completely topological in origin and describes the magnetic field of the monopole with a regular timelike potential. Using the field-theoretic description of monopoles in terms of a complex scalar field ϕ, the effective Lagrangian given by Eq. (3.5) yields the dual dynamics at the phenomenological level. The dynamical symmetry breaking induced by the effective potential (3.6) in the strong coupling limit leads to magnetic condensation of the dual RCD vacuum which is responsible for setting the confinement forces in confining any colored object. The resulting flux tube structure in dual RCD plays a major role in the confinement mechanism and has a close analogy with the dual Ginzburg–Landau theory. The vacuum solutions to the field equations have been obtained and the RCD superconducting medium is characterized by the value of the order parameter $|\phi| = \phi_0$ [the classical vacuum value of the field $\phi(x)$] which signals the breaking of the associated magnetic symmetry dynamically. The finite-energy solutions constructed in the asymptotic limit have the correct exponentially decaying behavior leading to depth or penetration of the color electric field in the dual RCD vacuum as $[(4\pi/g\sqrt{2}\phi_0]^{-1}$, which has important bearings on the nature of the type of superconducting dual RCD vacuum [20].

The calculation of finite energy per string length for large ρ and its analysis in terms of the linear and logarithmic nature (when the quarks are very distant) has been shown to have a close relationship with the color confinement, which is in agreement with the results of other authors [3, 13, 25, 26]. The resulting flux tube energy expression clearly demonstrates the emergence of a confinement–deconfinement phase transition in the dual QCD vacuum. The energy expression confirms that the large-distance response of the magnetically condensed dual QCD vacuum is of a strongly confining nature and is responsible for the disappearance of the quarks and monopoles from the physical spectrum of the theory at large hadronic distances. The flux tube energy and the resulting confining potential have several phenomenological implications for the mass spectrum of the physical states including the possibility of a multi-flux-tube structure of the superconducting dual QCD vacuum, which shall be dealt with in future work.

ACKNOWLEDGMENTS

One of the authors (H.C.P.) thanks the Council of Scientific and Industrial Research (CSIR), New Delhi, India, for financial assistance during the course of this work.

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